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Magnetized LRS Bianchi Type-I Massive String Cosmological Model for Perfect Fluid Distribution with Cosmological Term

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Abstract: In the present paper, we have investigated magnetized LRS Bianchi type-I massive string cosmological model for perfect fluid distribution with cosmological term Λ . To obtain the explicit solution of the model we suppose expansion θ is propotional to the shear σ and Λ is inversely proportional to R^3 , where R is scale factor under two cases: (i) $\rho + \lambda = 0$ and (ii) $\rho - \lambda = 0$ where ρ and λ are proper energy density and string tension density respectively. We have also discussed physical and geometrical characteristics of the cosmological model.

*Keywords***:** Bianchi type-I, perfect fluid, electromagnetic field, cosmological term, massive string.

I. INTRODUCTION

Einstein and De-Sitter space time, which are static universe, are particular case of Bianchi type-I cosmological models. These models are the natural generalization of FRW models with zero curvature. Bali and Singh [2, 3], Deo et al. [7], Dubey et al. [9], Dwivedi and Tiwari [11], Singh and Tiwari [19] have studied locally rotationally symmetric Bianchi type-I cosmological models in various contexts. Anisotropic Bianchi type-I cosmological models are studied by Chawla et al. [5], Deo et al. [6], Pradhan et al. [16]. Many more investigations in this field have been carried out by number of researchers viz. Dwivedi et al. [10], Humad and Shrimali [12], Mathur et al. [14], Tiwari [20], Tripathi et al. [25].

The cosmological constant problem is one of the most prominent and interesting unsolved problems in cosmology. Many relativistic try to solve this problem by considering cosmological models with dynamic cosmological term Λ . Tiwari and Sharma [22] have investigated Bianchi type-I magnetic string cosmological model with bulk viscosity and time-dependent term Λ . Homogeneous Bianchi type-I cosmological model filled with viscous fluid with a varying Λ is considered by Kandalkar et al. [13]. Dubey et al. [8], Tiwari et al. [21] have studied Bianchi type-I cosmological model with Λ term in different contexts.

The study of cosmological models with electromagnetic field plays an important role for the evolution of galaxies and astral bodies. Bianchi type-I cosmological models with electromagnetic field are studied by many researchers namely, Bali and Anjali [1], Mathur et al. [15], Saha et al. [18], Tripathi et al. [24]. Magnetized bulk viscous fluid cosmological models with time varying cosmological term is considered by Pawar and Shahare [17]. Bhoyar and Chirde [4] studied magnetized anti-stiff fluid cosmological models with variable cosmological constant. Tiwari et al. [23] have investigated Bianchi type-IX cosmological model with varying lambda term. Tyagi et al. [26] have studied Bianchi type-IX string cosmological models for perfect fluid distribution.

Motivating by the above discussions, in the present paper, we have investigated LRS Bianchi type-I massive string cosmological model for perfect fluid distribution with electromagnetic field and cosmological term Λ . To obtain the explicit solution of the model we suppose expansion θ is propotional to the shear σ and Λ is inversely proportional to R^3 , where R is scale factor under two cases: (i) $\rho + \lambda = 0$ and (ii) $\rho - \lambda = 0$ where ρ and λ are proper energy density and string tension density respectively. We have also discussed physical and geometrical characteristics of the cosmological model.

II. METRIC AND FIELD EQUATIONS

The line element for Bianchi type-I space-time is considered as $ds^2 = -dt^2 + A^2 dx^2 + B^2(d$) (1)

where A and B depends on cosmic time t only.

The energy momentum tensor T_i^j for a cloud of strings with magnetic field in presence of perfect fluid is given by

The Field Equations (12) - (14) are arrangement of three equations with six unknown parameters A, B, Λ , λ , ρ , p. Thus initially the system is undetermined, so we need three more equations to attain the complete solution of the system. Therefore we suppose that expansion θ is proportional to shear σ (15)

i.e.
$$
\Lambda = \frac{a}{R^3} = \frac{a}{AB^2}
$$

We assume the above conditions under two cases: (i) $\rho + \lambda = 0$ and (ii) $\rho - \lambda = 0$. **Case I**. $\rho + \lambda = 0$ (17) On adding equations (12) and (14), we obtain

$$
\frac{2B_{44}}{B} + \frac{2B_4^2}{B^2} + \frac{2A_4B_4}{AB} + 2\Lambda = \lambda + \rho - p + \frac{K}{B^4}
$$
\n(18)

where $\frac{\bar{H}^2}{\equiv}$ $\frac{1}{\overline{\mu}}$ =

Using equations (13) , (15) , (16) and (17) in equation (18) , we get $2(2n+2-n^2)B_4^2$ $3K$ 2α

$$
2B_{44} + \frac{-(1+i)(1+i)(1+i)}{(1-i)(1-i)(1-i)} = \frac{2i}{(1-i)(1-i)(1-i)} - \frac{2i}{(1-i)(1-i)(1-i)}
$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (19), we obtain

 (16)

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$$
\frac{df^2}{dB} + \frac{2(2n+2-n^2)f^2}{(1-n)B} = \frac{3K}{(1-n)B^3} - \frac{2\alpha}{(1-n)B^{n+1}}
$$
\nOn integration, equation (20) leads to

$$
f^{2} = \frac{2\alpha}{(n^{2} - 3n - 4)B^{n}} - \frac{3K}{2(n^{2} - 3n - 1)B^{2}} + \frac{l}{B^{\frac{2(2n + 2 - n^{2})}{1 - n}}}
$$
(21)

here l is the integrating constant.

 ds^2

From equation (21), we get\n
$$
\int_{R}^{R}
$$

$$
\int \frac{2a}{\sqrt{(n^2 - 3n - 4)B^n} - \frac{3K}{2(n^2 - 3n - 1)B^2} + \frac{l}{B^{\frac{2(2n + 2 - n^2)}{1 - n}}}} = t + M
$$
\n(22)

where M is the integrating constant. Value of B can be obtained from equation (22).

Hence, by appropriate transformation of coordinates i.e. $B=T$, $x=X$, $y=Y$ and $z=Z$, metric (1) becomes

$$
= -\frac{dT^2}{\left[\frac{2\alpha}{(n^2 - 3n - 4)T^n} - \frac{3K}{2(n^2 - 3n - 1)T^2} + \frac{l}{r^{2(2n + 2 - n^2)}}\right]} + T^{2n}d + T^2(dY^2 + dZ^2)
$$
(23)

Case II.
$$
\rho - \lambda = 0
$$
 (24)
On subtracting equation (12) from (14), we obtain

$$
\frac{2A_4B_4}{AB} - \frac{2B_{44}}{B} = \rho - \lambda + p
$$
\nUsing equations (13), (15), (16) and (24) in equation (25), we get

\n
$$
\frac{2A_4B_4}{AB} = \frac{2B_{44}}{B} = \frac{2B_{44}}{B}
$$
\n(25)

$$
2B_{44} - \frac{2n(n+2)B_4^2}{(1-n)B} = \frac{K}{(1-n)B^3} + \frac{2\alpha}{(1-n)B^{n+1}}
$$
(26)

On putting
$$
B_4 = f(B)
$$
 and $B_{44} = ff'$ in equation (26), we obtain
\n
$$
\frac{df^2}{dB} - \frac{2n(n+2)f^2}{(1-n)B} = \frac{K}{(1-n)B^3} + \frac{2\alpha}{(1-n)B^{n+1}}
$$
\n(27)

$$
dB \t(1-n)B \t(1-n)B^{3} \t(1-n)B^{n+1}
$$

On integration, equation (27) leads to

$$
f^{2} = -\frac{K}{\sqrt{2\pi}} - \frac{2\alpha}{\sqrt{2\pi}} + \frac{C}{\sqrt{2\pi}}
$$
 (28)

$$
f^{2} = -\frac{1}{2(n^{2} + n + 1)B^{2}} - \frac{1}{n(n + 5)B^{n}} + \frac{1}{B(n+1)}
$$
(28)

here C is the integrating constant.

From equation (28), we get

$$
\int \frac{dB}{\sqrt{-\frac{K}{2(n^2+n+1)B^2} - \frac{2\alpha}{n(n+5)B^n} + \frac{C}{B^{\frac{2n(n+2)}{n-1}}}}}} = t + N
$$
\n(29)

where N is the integrating constant. Value of B can be obtained from equation (29).

Hence, by appropriate transformation of coordinates i.e. $B=T$, $x=X$, $y=Y$ and $z=Z$, metric (1) becomes

$$
ds^{2} = -\frac{dT^{2}}{\left[-\frac{2\alpha}{n(n+5)T^{n}} - \frac{K}{2(n^{2}+n+1)T^{2}} + \frac{C}{T^{2n(n+2)}}\right]} + T^{2n}dX^{2} + T^{2}(dY^{2} + dZ^{2})
$$
\n(30)

IV. PHYSICAL AND GEOMETRICAL CHARACTERISTICS

For the model (23), energy density (ρ), string tension density (λ), particle energy density (ρ_p), pressure (p), expansion(θ), shear (σ) are given by

$$
\rho = -\lambda = \frac{(n^2 + n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} - \frac{(n^2 + 3n + 2)K}{2(n^2 - 3n - 1)T^4} + \frac{l(2n + 1)}{T^{2(n^2 - n - 3)\over n - 1}}
$$
(31)

$$
\rho_p = \frac{2(n^2 + n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} - \frac{(n^2 + 3n + 2)K}{(n^2 - 3n - 1)T^4} + \frac{2l(2n + 1)}{r^{2(n^2 - n - 3)}}
$$
(32)

$$
p = -\frac{2(n^2 - 2n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{(n^2 - 1)K}{(n^2 - 3n - 1)T^4} + \frac{2l(2n + 1)}{(1 - n)T^{2(n^2 - n - 3)}}
$$
(33)

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$$
\theta = (n+2) \left[\frac{2\alpha}{(n^2 - 3n - 4)T^{n+2}} - \frac{3K}{2(n^2 - 3n - 1)T^4} + \frac{l}{T^{2(n^2 - n - 3)} \choose n-1} \right]^{\frac{1}{2}}
$$
(34)

$$
\sigma = \frac{(n-1)}{\sqrt{3}} \left[\frac{2\alpha}{(n^2 - 3n - 4)T^{n+2}} - \frac{3K}{2(n^2 - 3n - 1)T^4} + \frac{l}{T^{\frac{2(n^2 - n - 3)}{n - 1}}} \right]^{\frac{1}{2}}
$$
(35)

For the model (30), energy density (ρ), string tension density (λ), particle energy density (ρ_p), pressure (p), expansion(θ), shear (σ) are given by

$$
\rho = \lambda = \frac{(n^2 + n - 2)\alpha}{n(n+5)T^{n+2}} - \frac{(n^2 + 3n + 2)K}{2(n^2 + n + 1)T^4} + \frac{C(2n+1)}{T^{2(n^2 + 3n - 1)\over n - 1}}
$$
(36)

$$
\rho_p = 0 \tag{37}
$$

$$
p = -\frac{6\alpha}{(n+5)T^{n+2}} - \frac{(n+1)K}{(n^2+n+1)T^4} + \frac{2Cn(2n+1)}{(n-1)T^{2(n^2+3n-1)}(n+1)T^{n+2}} \tag{38}
$$

$$
\theta = (n+2) \left[-\frac{2\alpha}{n(n+5)T^{n+2}} - \frac{K}{2(n^2+n+1)T^4} + \frac{C}{T^{\frac{2(n^2+3n-1)}{n-1}}} \right]^{\frac{1}{2}}
$$
(39)

$$
\sigma = \frac{(n-1)}{\sqrt{3}} \left[-\frac{2\alpha}{n(n+5)T^{n+2}} - \frac{K}{2(n^2+n+1)T^4} + \frac{C}{T^{2(n^2+3n-1)} \overline{n-1}} \right]^{\frac{1}{2}}
$$
(40)

Also for both models (23) and (30) cosmological term Λ is given by $\Lambda = \frac{\alpha}{\pi n}$ $Tⁿ$ (41)

V. SOLUTION IN THE ABSENCE OF MAGNETIC FIELD

Case I. $\rho + \lambda = 0$

In the absence of magnetic field, metric (23) becomes

$$
ds^{2} = -\frac{dT^{2}}{\left[\frac{2\alpha}{(n^{2} - 3n - 4)T^{n}} + \frac{l}{T^{\frac{2(2n + 2 - n^{2})}{1 - n}}}\right]} + T^{2n}dX^{2} + T^{2}(dY^{2} + dZ^{2})
$$
\n(42)

For the model (42), energy density (ρ), string tension density (λ), particle energy density (ρ_p), pressure (p), expansion(θ), shear (σ) are given by

$$
\rho = -\lambda = \frac{(n^2 + n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{l(2n + 1)}{T^{\frac{2(n^2 - n - 3)}{n - 1}}} \tag{43}
$$

$$
\rho_p = \frac{2(n^2 + n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{2l(2n + 1)}{T^{\frac{2(n^2 - n - 3)}{n-1}}} \tag{44}
$$

$$
p = -\frac{2(n^2 - 2n - 2)\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{2l(2n + 1)}{(1 - n)T^{\frac{2(n^2 - n - 3)}{n - 1}}}
$$
(45)

$$
\theta = (n+2) \left[\frac{2\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{l}{T^{\frac{2(n^2 - n - 3)}{n-1}}} \right]_1^{\frac{1}{2}}
$$
(46)

$$
\sigma = \frac{(n-1)}{\sqrt{3}} \left[\frac{2\alpha}{(n^2 - 3n - 4)T^{n+2}} + \frac{l}{T^{\frac{2(n^2 - n - 3)}{n-1}}} \right]^{\frac{1}{2}}
$$
(47)

Case II. $\rho - \lambda = 0$

In the absence of magnetic field, metric (30) becomes

$$
ds^{2} = -\frac{dT^{2}}{\left[-\frac{2\alpha}{n(n+5)T^{n}} + \frac{C}{T^{\frac{2n(n+2)}{n-1}}}\right]} + T^{2n}dX^{2} + T^{2}(dY^{2} + dZ^{2})
$$
\n(48)

For the model (48), energy density (ρ), string tension density (λ), particle energy density (ρ_n) , pressure (p), expansion(θ), shear (σ) are given by

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$$
\rho = \lambda = \frac{(n^2 + n - 2)\alpha}{n(n+5)T^{n+2}} + \frac{C(2n+1)}{T^{2(n^2+3n-1)\over n-1}}
$$
(49)

$$
\rho_p = 0 \tag{50}
$$
\n
$$
\alpha \quad 2Cn(2n+1) \tag{51}
$$

$$
p = -\frac{1}{(n+5)T^{n+2}} + \frac{1}{(n-1)T^{\frac{2(n^2+3n-1)}{n-1}}}
$$
(51)

$$
\theta = (n+2) \left[-\frac{2\alpha}{n(n+5)T^{n+2}} + \frac{C}{T^{\frac{2(n^2+3n-1)}{n-1}}} \right]^{\frac{1}{2}}
$$
(52)

$$
\sigma = \frac{(n-1)}{\sqrt{3}} \left[-\frac{2\alpha}{n(n+5)T^{n+2}} + \frac{C}{T^{\frac{2(n^2+3n-1)}{n-1}}} \right]^{\frac{1}{2}}
$$
(53)

VI. CONCLUSION

The models (23) and (30) start expanding with big bang at $T = 0$. The expansion θ is decreasing function of cosmic time T for n>0 and approaches to zero as $T \to \infty$. We also observe that it stops when $n = -2$. Since $T \to \infty$, $\frac{\sigma}{a}$ $\frac{\sigma}{\theta} = \frac{n}{\sqrt{3}}$ $\frac{n-1}{\sqrt{3}(n+2)} \neq 0$, therefore the model does not approach isotropy for large values of T, however it is isotropized for $n = 1$.

Also, we can observe that energy density (ρ) , string tension density (λ) and pressure (p) for both models are decreasing function of time T for n>0 and approaches to zero as $T \to \infty$. We find similar type of conclusions for models (42) and (48) as for models (23) and (30).

Also, Cosmological term Λ for these models are found to be decreasing function of time T for n>-2 and it approaches to zero at late time, which is in agreement with present astronomical observations.

Therefore, in general, all models represent expanding, shearing and non-rotating universe.

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